A NUMERICAL APPROACH TO PROBLEM OF GENERALIZED PLANE STRESS

BY

TIE PAO TUNG

B.S., Chiao Tung University, 1939
M.S., Purdue University, 1947

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ENGINEERING
IN THE GRADUATE COLLEGE OF THE
UNIVERSITY OF ILLINOIS, 1950

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R. UNIV. BIBLIOTHEEK LEIDEN

A NUMERICAL APPROACH TO PROBLEM OF GENERALIZED PLANE STRESS

Introduction

The object of this thesis was the development of a numerical procedure for the problems of the generalized plane stress of the theory of elasticity. A correction solution was found and then added to the two dimensional plane strain solution so that the corrected three dimensional solution satisfied all the necessary equations. The method of finite differences was used to obtain the correction solution and the displacements were used as a working medium to satisfy the analogous difference equations of the equations of equilibrium and boundary conditions. The method of relaxation was used to obtain the solution of the resulting simultaneous algebraic equations.

A Test Problem

An infinite plate with a circular cylindrical hole with its diameter equal to the thickness of the plate, stressed under uniaxial tension, was taken as a test problem and was solved in three different set-ups: (1) in polar cylindrical coordinates; (2) in polar cylindrical coordinates, with the use of the higher order differences; (3) in rectangular coordinates. The resulting stresses were shown in the attached table with the exact solutions.

Polar Cylindrical Coordinates

By taking the sinusoidal variation of the stresses and the displacements around any circumference, the three dimensional correction solution of the test problem was reduced to a two dimensional problem in radial slices. A number of different sizes of mesh were used corresponding to 8, 16, and 32 divisions in the thickness of the plate. Each finer net in general was

carried out for a radial distance of only six mesh length, the boundary values at this point being taken from the preceeding solution for coarser net.

By including the higher order terms in the finite differences the result could be improved even in a fairly large mesh size. In the second set-up, the finite differences including the higher order terms up to the 4th order were used in the coarse net, 8 divisions in the thickness of the plate.

Rectangular Coordinates

Three dimensional rectangular network of cubes was laid in the region of the test problem. There were 4 divisions in the thickness of the plate and 4 in the radial direction. The displacements at the fictitious nodes outside the curved boundary of the hole were made multiple-valued in order to fit the curved boundary in the rectangular network to a better approximation. The method of relaxation did not work quite well, because the formulation of the boundary equations on the curved boundary usually gives unfavorable arrangement of numerical coefficients.

SUMMARY OF RESULTS OF NUMERICAL SOLUTIONS
STRESSES AT EDGE OF HOLE
T/D = 1

Method	λ/T	Max. σ_z	Max.	
		at mid-plane	at mid-plane	at edge
in Polar Cylindrical Coordinates	1/8	0.241	3.157	2.80
	1/16	0.256	3.162	2.811
	1/32	0.258	3.163	2.811
	1/8*	0.270	3.130	2.791
in Rectangular Coordinates	1/4	0.282	2.954	2.717
Green		0.268	3.100	2.81
Sadowsky-Sternberg (interpolated from curves)		0.347	3.08	2.79

* Including the higher order differences.
D = Diameter of hole
T = Thickness of the Plate
λ = Length of mesh for numerical solution
μ = Poisson's ratio, taken as 1/4

VITA

Tie Pao Tung was born in Peiking, China, August, 17, 1916. After he received the degree of B.Sc.C.E. from Chiao Tung University, July, 1939, he was employed by the Bridge Design Office, Ministry of Communication, 1939-1945.

He was sent to the States in 1946 to work in the highway laboratory of the Joint Highway Research Project, Purdue University, as a graduate assistant. He received the degree of M.Sc.C.E. in February, 1947. Afterwards, he entered the University of Illinois. In June, 1948, he was appointed the Special Research Assistant of Civil Engineering, the position of which he now holds.